Total No. of printed pages = 10

RETEST EXAMINATION - 2019

Semester : 1st (Old)

Subject Code : Sc - 201

MATHEMATICS - I

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

Marks – 25

Answer *all* the questions from PART – A.

1. Choose the correct answers from the four options provided for each of the following : 1×10=10

(a) The value of $\log_3 \log_3 27$ is

(i) 16 (ii) 8

(iii) 4 (iv) 1

(b) If ω is an imaginary cube root of unity the $(1+\omega)(1+\omega^2)$ is equal to

(i) -1 (ii) 0

(iii) 1 (iv) 2

(c) The roots of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are imaginary numbers if

(i) $b^2 - 4ac < 0$ (ii) $b^2 - 4ac > 0$

(ii) $b^2 - 4ac = 0$ (iv) $c = 0$

(d) The value of the determinant $\begin{vmatrix} 54 & 72 & 143 \\ 48 & 64 & 239 \\ 42 & 56 & 378 \end{vmatrix}$ is

(i) 7 (ii) 10

(iii) 41 (iv) 0

(e) If ${}^{n}C_3 = {}^{n}C_{17}$, then the value of n is

(i) 20 (ii) 24

(ii) 22 (iv) 23

(f) Modulus of the Complex Number 1+i is

(i) 2 (ii) $\sqrt{2}$

(iii) 1 (iv) 3

(g) If $x \propto y^2$, $x = 9$ when $y = 6$,then $y = 4$ implies x is equal to

(i) 4 (ii) 25

(iii) 10 (iv) 1

(h) The argument of the complex number $1 + i$ is

(i) 30° (ii) 45°

(iii) 60° (iv) 90°

(i) The 6^{th} term of the Geometric Progression $-6, 12, -24, \ldots\ldots$ is

(i) -32 (ii) 32

(iii) 192 (iv) -192

(j) The number of terms in the expansion of $(2x + 3y)^{10}$ is

(i) 10 (ii) 5

(iii) 6 (iv) 11

2. Write true or false : $1\times5=5$

(a) The sum of two conjugate complex number is a real number.

(b) If $c = a$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are reciprocal to each other.

(c) If A and G are the Arithmetic and Geometric mean respectively between two numbers then $A < G$.

(d) ${}^{n}C_{r} = r!{}^{n}P_{r}$ $(r < n)$.

(e) If elements of any two rows or columns of a determinant are identical, then the value of the determinant will be zero.

3. Choose the correct answers : $1\times5=5$

(a) The value of 570° is

(i) $\frac{1}{\sqrt{3}}$ (ii) -1

(iii) $\sqrt{3}$ (iv) 0

(b) If $\tan A = \frac{3}{4}$, the value of $\tan 2A$ is

(i) $\frac{24}{7}$ (ii) $\frac{7}{25}$

(iii) $\frac{24}{25}$ (iv) $\frac{7}{24}$

(c) $\cos^2 38^\circ + \cos^2 52^\circ$ is equal to

(i) $\frac{1}{\sqrt{2}}$ (ii) 1

(iii) $\frac{1}{2}$ (iv) 0

(d) $\frac{1-\cos 2\theta}{\sin 2\theta}$ is equal to

(i) $\tan\theta$ (ii) $\cot\theta$

(iii) $\sin\theta$ (iv) $\cos\theta$

(e) If $2\sin^2\theta = 1$ $(0 \leq \theta \leq 90^\circ)$, then the value of θ is

(i) 90° (ii) 60°

(iii) 30° (iv) 45°

4. Fill in the blanks : 1×5=5

(a) The area of a hexagon with side length 8 cm is _____.

(b) The volume of a sphere of radius 6 cm is _____.

(c) The volume of a Cone having radius of the base 'a' unit and height 'h' is _____.

(d) The volume of a cylinder having radius of the base 3 cm and height 4cm is _____.

(e) The slant height of a cone with radius 3 cm and height 4 cm is _____.

PART – B

Marks – 45

Answer *all* the questions of PART – B.

4. Answer any *five* questions : 2×5=10

(i) Prove that $\log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx = 2$.

(ii) Reduce $\frac{5+3i}{(1+i)(3+2i)}$ into A + iB form.

(iii) If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

(iv) Find the number of arrangements of the letters of the word EDUCATION.

(v) Find 9^{th} terms of the Expansion $(x+2y)^{18}$.

(vi) In how many ways a committee of 4 boys and 3 girls can be formed from 10 boys and 8 girls ?

(vii) If $(n+1)_{p_3} = 4.n_{p_2}$, Find n.

(viii) Find 4 Arithmetic means between 3 and 23.

(ix) If $x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$, Show that $z \propto x$.

(x) Find the Sum of $5+9+13+\ldots\ldots$ to 10^{th} term.

5. Answer any *three* questions : $3\times3=9$

(i) Show that

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)(ab+bc+ca-a^2-b^2-c^2)$$

(ii) Prove that $1+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots\infty=\frac{1}{2}(e+\frac{1}{e})$.

(iii) Find the co-efficient of x^6 in the expansion of $(x+\frac{1}{2x^2})^{12}$.

(iv) Solve by Cramer's rule : $x+y-z=1$; $x-y+2z=2$; $x+2y+z=4$.

(v) If the roots of the equation $lx^2+mx+n=0$ are in the ratio p : q, prove that,

$$\sqrt{\frac{p}{q}}\sqrt{\frac{q}{p}}\sqrt{\frac{m}{l}}=0.$$

(vi) Find three numbers in G.P whose sum is 13 and product is 27.

6. Answer any *five* questions : 2×5=10

(i) Find the value of $\cos1^\circ \cos2^\circ \cos3^\circ \ldots\ldots \cos100^\circ$.

(ii) Prove that $\cos^2 15^\circ + \cos^2 45^\circ + \cos^2 75^\circ = \frac{3}{2}$.

(iii) Prove that $\frac{1-\sin\theta}{\cos\theta}=\frac{\cos\theta}{1+\sin\theta}$

(iv) Prove that $\frac{\sin 5A+\sin 3A}{\cos 5a+\cos 3A} = \tan 4A$

(v) Show that $\cos^{-1}x = \sin^{-1}\sqrt{(1-x^2)}$.

(vi) Find the smallest angle of the triangle having side lengths 6 cm, 8 cm, and 10 cm.

(vii) Prove that, $(1+\tan 17^\circ)(1+\tan 28^\circ) = 2$.

7. Answer any *two* questions : $3\frac{1}{2}\times 2 = 7$

(i) If $A + B + C = \pi$, prove that $\sin 2A + \sin 2B - \sin 2C = 4\cos A \cos B \cos C$.

(ii) In any triangle ABC, prove that, $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$ a, b, c denote the sides BC, CA and AB of the triangle ABC.

(iii) Prove that $\sin 3A = 3\sin A - 4\sin^3 A$.

(iv) Solve for $2\cos^2\theta + \sin\theta - 1 = 0$ $(0 \le \theta \le 2\pi)$.

8. Answer any *three* questions : $3\times 3 = 9$

(i) Find the area of regular hexagon whose length of each side is 8 cm.

(ii) The volume of a triangular prism is 90 cu.cm and length of sides of bases are 12 cm, 13 cm and 5 cm. Find the height and lateral area of the prism.

(iii) A reservoir of depth 10mt is bounded by two rectangular faces of area 20×10 and 30×15 sq.mt. Find the amount of water the reservoir can hold.

(iv) The area of the whole surface of a right circular cone is 15sq.mt and the slant height is three times the radius of the base. Find the radius of the base.

(v) Find the volume of the solid. The cross-section (A) of a solid (area sq.cm) at a distance x cm from the end is as follows :

x :	10	30	50	70	90	110	130	150
A :	120	123	129	129	131	135	142	156